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SOCIAL SCIENCE WORKING PAPER 302

January 1980

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This paper analyzes strategic lobbying behavior under the assumption of disorganized opposition using an optimal control approach. Optimal rates of lobbying expenditure are shown to be increasing functions of time. A specific example is computed to determine the influence of various parameters. Next, the case of lobbying against an active counterlobby is considered in a game theoretic framework. The Nash equilibrium is computed for a specific game with symmetric players. The equilibrium rates of expenditure for the lobbyists may be either everywhere increasing, everywhere decreasing, or single peaked. In addition, some comparative statics results are reversed relative to those of the disorganized opposition model.

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I Introduction

Many decisions which affect the economic well-being of large numbers of people are made by relatively small committees of selected representatives. Examples include rate-setting by various regulatory bodies, minimum wage laws and taxation policy. These few people in positions of responsibility are then the natural targets of organizations engaged in lobbying either for or against a particular measure. In this paper, I attempt to model this phenomenon under alternative assumptions regarding the existence of an effective counterlobby. I consider first the case where there is no organized opposition, using an optimal control approach. Then I extend the model to include an active counterlobby, using a game theoretic analysis.

I make no particular assumptions regarding the nature of lobbying behavior. It may consist of information provision, campaign contributions or other compensation. I merely assume that lobbying effort can influence a particular committee member's beliefs and can (stochastically) elicit commitment from that member to the lobbyist's cause. While the committee member chooses whether or not to commit himself, I assume that, as far as the lobbyist is concerned, commitment is probabilistic. I assume that, once committed, the committee member cannot become uncommitted. For example, commitment may consist of making a public statement in support of the lobbyist's cause; I then assume that it would be politically inexpedient to renege.

While it makes little difference whether I denominate the value of the target's commitment in terms of utility to the lobbyist or in terms of expected profit, in light of the professionalism of most lobbying groups and the prevalence of contract lobbyists, I assume that the value of a particular committee member's commitment can be summarized in dollar terms. Then I propose expected profit maximization as the lobbyist's goal.

II Lobbying Against Disorganized Opposition

Consider first the case wherein the lobbying organization attempts to effect passage of a particular measure against disorganized opposition. By disorganized opposition I mean that no counterlobby exists. While this covers by no means all cases, it is representative of many. For example, most special interest lobbying is against disorganized opposition, for the losses are often diffused over a large population while the benefits accrue to a relative few. Even a small cost to organizing may preclude the formation of an effective counterlobby. Although there is no established opposition lobby, there may be a certain inertial force working against the measure which can be countered by lobbying effort. We refer to the lobbying organization or its paid representative as the <u>lobbyist</u>. The person or group being lobbied is the <u>target</u>.

The problem can be posed as one of optimal control. The control variable for the lobbyist is the rate of expenditure on lobbying until the date of the scheduled vote. If commitment is not obtained before the date of the vote, then the target is assumed to vote

for the lobbyist's cause with some probability, which may depend on accumulated lobbying effort.

T denotes the scheduled date of the vote or decision. With initial date 0, the planning horizon then becomes $[0,T]$. The rate of lobbying expenditure at t is $u(t)$. We assume that $u(t)$ is nonnegative and belongs to some set of admissible controls, U. Since we will not be concerned explicitly with the circumstances under which optimal controls exist, we leave the set of admissible controls unspecified for now.

The expenditure level $u(t)$ is assumed to create a stock of accumulated lobbying effort; $z(t)$ is accumulated lobbying effort at t. It is increased by lobbying expenditure $u(t)$ via the production function

$$\dot{z}(t) = g(u(t),z(t))$$

starting from some initial stock $z(0) = z_o \geq 0$. We assume that $g(u,z) \geq 0$ and $g(0,z) = 0 \ \forall \ z$. That is, nonnegative lobbying expenditures produce nonnegative increments to z, but no free increments can be had. There is no exogenous growth or decay in accumulated lobbying effort. We add that $g(u,z)$ is C^2 on $\mathbb{R}^2_+$ with $g_1 > 0$, $g_{11} < 0$; decreasing returns to lobbying expenditure.

If we choose as our admissible set

$$U = \{u(t): u(t) \geq 0, \ u(t) \text{ continuous}, \ \forall \ t \in [0,T]\}$$

then $g(u(t),z)$ is continuous in t and C^2 in z on $[0,T] \times \mathbb{R}^+$ and the differential equation

$$\dot{z} = g(u(t),z)$$

with initial condition $z(0) = z_o \geq 0$ has a unique C^1 solution $z(t)$ through z_o, for any $u(t) \in U$.

Whether or not the target commits itself to vote for the lobbyist's cause depends upon how much sympathy the target has for that cause. I assume that lobbying effort can enhance that sympathy, perhaps only stochastically. Since the lobbyist really can't tell how sympathetic the target is, or exactly how its lobbying effort affects this sympathy, I assume that it must rely on its subjective assessments of target behavior. I assume that the lobbyist's beliefs regarding how much accumulated lobbying effort is required (to stimulate sufficient sympathy) to elicit commitment are summarized in the subjective probability distribution

$$F(z) \equiv \Pr\{\text{target commits with accumulated lobbying effort} \leq z\}.$$

This need not be a proper distribution. That is, there may be a nonzero probability that no finite amount of accumulated lobbying effort is sufficient to induce a target to commit itself.

Since $z(t)$ is nondecreasing, this induces a distribution over the random date of commitment, τ:

$$\Pr\{\tau \leq t\} = F(z(t)), \quad t \in [0,T].$$

Denote the value of the target's commitment by P. This value is independent of when commitment is obtained (so long as it is obtained by or at T), since the measure will not take effect until T. For example, if the measure will result in some positive wealth transfer to the members of the lobbying organization, then P denotes the present value of the wealth transfer (if the target is the sole decision-maker) or some expected value weighted by the degree of influence of the target over the decision.

The rate used to discount the stream of future lobbying expenditures will be r. If no commitment is obtained by T, there remains a probability that the target will vote in the lobbyist's favor. Denote this probability, which may depend on accumulated lobbying effort, by $\rho(z(T))$.

We can formulate the lobbyist's payoff as follows:

$$J(u) = PF(z(T)) + P\rho(z(T))(1 - F(z(T))) - \int_0^T e^{-rt}(1 - F(z(t)))u(t)dt. \tag{1}$$

That is, the lobbyist receives the value P if either a) it obtains the target's commitment by T, which occurs with probability $F(z(T))$; or b) it fails to obtain commitment by T, but the target votes or decides in favor of the lobbyist at T. This occurs with probability $(1 - F(z(T)))\rho(z(T))$.

Expenditures $u(t)$ are made only until commitment is obtained, due to the irrevocability assumption. Thus costs accrue at the (expected) rate of $(1 - F(z(t)))u(t)$ and are discounted at the rate r.

The lobbyist's goal, then, is to choose a lobbying expenditure path u(t) so as to maximize expression (1) subject to the constraints

$$\begin{aligned} &\dot{z}(t) = g(u,z),\ z(0) = z_o \geq 0 \\ &u(t) \text{ admissible} \end{aligned} \tag{2}$$

Proposition 1: Suppose F(z) has a nonnegative density, $g_2 \leq 0$ and $g_{12} \geq 0$. If $u^*(t)$ exists, is positive, and is differentiable $\forall\ t \in [0,T]$, then

$u^*(t)$ is an increasing function of t.

Remark: We are concerned only with those cases where commitment has not yet been obtained $(F(z(t)) < 1)$. If commitment has been obtained, then $u^*(t) = 0\ \forall\ t$ after the commitment date.

Proof: The Hamiltonian is

$$H = -e^{-rt}(1 - F(z))u + q(t)g(u,z)$$

where q(t) is the costate variable. If $u^*(t)$ exists, it must satisfy the usual conditions for optimality:

$$-e^{-rt}(1 - F(z)) + q(t)\ g_1(u^*,z) = 0, \tag{3}$$

$$\dot{q} = -e^{-rt}\ F'(z)u^* - qg_2(u^*,z), \tag{4}$$

$$q(T) = \frac{\partial}{\partial z}\ [PF(z(T)) + P\rho(z(T))(1 - F(z(T)))].$$

Solving equation (3) for q(t) yields

$$q(t) = \frac{e^{-rt}(1 - F(z))}{g_1(u_1^*,z)} \tag{5}$$

Differentiating (5) and equating the resulting expression for $\dot{q}$ to that of equation (4) yields the following:

$$(1 - F)g_{11}\dot{u}^* = F'g\ [u^*g_1 - g] + (1 - F)[g_1g_2 - rg_1 - gg_{12}].$$

Since $g_{11} < 0$ and $1 - F > 0$, $\text{sgn}\ \dot{u} = -\text{sgn}\ [F'g_1(u^*g_1 - g) + (1 - F)(g_1g_2 - rg_1 - gg_{12})]$. Since $g(0,z) = 0$ and $g(u,z)$ is strictly concave in u for fixed z, $[g_1u^* - g] < 0$. Assuming $g_2 \leq 0$ and $g_{12} \geq 0$ implies that the second term is also negative. Thus $u^*(t)$ is strictly increasing. Consider the restrictions $g_2 \leq 0$, $g_{12} \geq 0$. They imply that it becomes no easier to create additional lobbying effort as z increases (and if it becomes more difficult, it does so at an increasing rate). In particular, what this rules out is a phenomenon akin to "learning by doing." That is, the more effort accumulated, the more productive is current expenditure. If we allow this phenomenon, then we are unable to determine sgn $\dot{u}^*$ unambiguously. With learning by doing, the lobbyist may be able to decrease expenditures as time passes since these expenditures become more and more productive.

In order to make any statements regarding the dependence of the optimal lobbying path on the various parameters of the model, we would need to examine a particular example. This is done in the next section.

III Example for the Unopposed Lobbyist

Consider the expression $h(z) = F'(z)/(1 - F(z))$. Then

$$h(z) = \Pr\{\text{commitment occurs in } (z, z+dz) \mid \text{commitment occurs after } z\}$$

We can argue almost equally convincingly for $h'(z) \geq 0$ and $h'(z) \leq 0$.

If $h'(z) > 0$, then imminent commitment becomes more and more likely as accumulated lobbying effort increases (given that commitment has not yet been obtained). If $h'(z) < 0$, then the lobbyist becomes increasingly convinced that commitment is still a long way off (given that commitment has not yet been obtained). We will assume the intermediate case of $h'(z) = 0$. That is, the lobbyist believes that each increment to accumulated effort is equally likely to elicit commitment (given no prior commitment). This corresponds to the distribution $F(z) = 1 - e^{-\lambda z}$, $\lambda \in \mathbb{R}^+$. In addition, we assume that

1) $g(u,z) = u^\gamma$, $\gamma \in (0,1)$
2) $\rho(z(T)) = \rho$, $\rho \in [0,1]$.

We will find it convenient to use a dynamic programming approach to solve the example. Define the value function

$$V(t,z) \equiv \underset{u \in U}{\text{maximum}} \int_t^T -e^{-rs} e^{-\lambda z(s)} u(s)\,ds + P(1 - e^{-\lambda z(T)}) + P\rho e^{-\lambda z(T)}$$

subject to $\dot{z} = u^\gamma$, $z(0) = 0$.

The Bellman equation of dynamic programming is

$$0 = V_t + \max_{u(t)} \; [V_z u^\gamma - e^{-rt} e^{-\lambda z} u]. \tag{6}$$

The maximizing rate of lobbying expenditure in feedback form is

$$u^o(t,z,V_z) = [\gamma V_z e^{rt} e^{\lambda z}]^{\frac{1}{1-\gamma}}.$$

The Hamilton-Jacobi equation

$$0 = V_t + V_z^{\frac{1}{\gamma - 1}} (e^{rt} e^{\lambda z})^{\frac{1}{1-\gamma}} \gamma^{\frac{1}{1-\gamma}} \tag{7}$$

with terminal conditions

$$V(T,z(T)) = P + P(\rho - 1)e^{-\lambda z(T)} \tag{8}$$

has solution

$$V(t,z) = P + \left[[P(\rho - 1)]^{-\alpha} + (-\lambda\gamma)^{\alpha+1} \frac{\left(e^{r\alpha t} - e^{r\alpha T}\right)}{r\alpha} \right]^{-\frac{1}{\alpha}} e^{-\lambda z}$$

where $\alpha = \frac{\gamma}{1-\gamma}$. Since $V(t,z)$ is C^1 and since $V(t,z)$ and $u^o = [\lambda V_z e^{rt} e^{\ z}]^{1/1(1-\lambda)}$ solve (6) and (8),

$$u^*(t) = [\gamma V_z(t,z) e^{rt} e^{\lambda z}]^{\frac{1}{1-\gamma}}$$

is a Nash equilibrium in pure strategies [Stalford and Leitmann (1973, Theorem 1)].

For simplicity, consider the case $\gamma = 1/2$. Then

$$u^*(t) = \left[\frac{2P(1-\rho)\lambda e^{rt}}{4 - P(1-\rho)\lambda^2 \frac{(e^{rt} - e^{rT})}{r}} \right]^2,$$

and the value of lobbying is

$$V(0,0) - P\rho = \frac{P^2(1-\rho)^2\lambda^2\gamma^2 \frac{(e^{rT}-1)}{r}}{1 - P(1-\rho)\lambda^2\gamma^2 \frac{(1-e^{rT})}{r}} \geq 0.$$

(We subtract $P\rho$ since this is the expected payoff without lobbying). For any $\rho < 1$, it pays to invest in lobbying at a positive rate.

The following proposition summarizes the comparative dynamics results.

Proposition 2: For $\rho < 1$,

i) $\frac{\partial u^*}{\partial P} > 0 \quad \forall\ t \in [0,T]$.

ii) $\frac{\partial u^*}{\partial \rho} < 0 \quad \forall\ t \in [0,T]$.

iii) $\frac{\partial u^*}{\partial T} < 0 \quad \forall\ t \in [0,T]$.

iv) $\frac{\partial u^*}{\partial \lambda} \gtrless 0$ as $t \gtrless \hat{t} = \frac{1}{r} \ln\left\{ e^{rT} - \frac{4r}{P(1-\rho)\lambda^2} \right\}$.

v) $\frac{\partial u^*}{\partial r} \gtrless 0$ as $t \gtrless \tilde{t}$, where $\tilde{t}$ solves

$$4T + \frac{P(1-\rho)\lambda^2}{r}\left[e^{rt}(t-T) - \frac{(e^{rt} - e^{rT})}{r} \right] = 0.$$

The proof of proposition 2, largely algebraic, is contained in the Appendix.

These results are readily interpretable. As we would expect, an increase in the value of commitment P stimulates an increase in the rate of lobbying expenditures over the entire planning horizon.

An increase in ρ, the probability of a favorable vote at T (given no prior commitment), results in a uniformly lower rate of investment in lobbying. This is reasonable since it is then less imperative that commitment be obtained before T. If the scheduled vote date T is pushed further into the future, then lobbying is pursued at a lower rate over the longer horizon, due to the concavity of the effort production function. Notice that $\hat{t}$ in part (iv) may be negative. If so, then $\frac{\partial u^*}{\partial \lambda} > 0 \quad \forall\ t \in [0,T]$. But if $\hat{t} > 0$, then an increase in λ results in a reallocation of resources away from the relatively near future and toward the relatively distant future. A similar redistribution of expenditure is implied by an increase in the discount rate r.

IV Lobbying Against Organized Opposition

Consider now the case where there is an active counterlobby. I model this counterlobby's goal as the maintenance of the status quo, while the lobby's goal is passage of a measure which changes the existing statutes, presumably to the benefit of the members of the lobbying organization. Now the lobbyist must take cognizance of the fact that the counterlobby will react to the lobbyist's actions and vice versa. I model this case as a dynamic game under the presumption that lobbying behavior is relatively easy to monitor due to disclosure laws and scrutiny by the press and various ethics and self-policing committees. Label the lobbyist player 1 and the counterlobbyist player 2. Again I assume that the target is a passive player in the game, being influenced in a systematic way by the efforts of the lobbyist and the counterlobbyist.

Now the target has 3 options. It may commit itself to the lobby, to the counterlobby, or to neither. Both lobby and counterlobby then attempt to influence the target so as to obtain a commitment for their particular side of the issue.

We modify our notation to include the counterlobby. Let $z_i(t)$ be player i's level of accumulated lobbying effort; player i can add to $z_i(t)$ by spending money on lobbying:

$$z_i = g^i(u_i, z_i)$$

where u_i is player i's rate of lobbying expenditure. I again assume that there are decreasing returns ($g_i > 0, g_{ii} < 0$) and that there is no exogenous growth or decay in effort: $g^i(0, z_i) = 0$.

Since we will not be explicitly concerned with existence problems, we take as i's strategy space, U_i, any set of reasonably nice functions of (t,z). For example,

$$U_i = \{u_i(t,z)\colon u_i(t,z) \geq 0 \text{ and } u_i(t,z) \text{ is continuous in } (t,z) \text{ and bounded and Lipschitz in } z, \ \forall\ (t,z) \in [0,T] \times \mathbb{R}_+^2\}.$$

Then the system

$$\dot{z}_1 = g^1(u_1(t,z_1,z_2),z_1), \quad z_1(0) = 0$$
$$\dot{z}_2 = g^2(u_2(t,z_1,z_2),z_2), \quad z_2(0) = 0$$

has a unique C^1 solution for every pair $(u_1,u_2) \in U_1 \times U_2$. That is, we are looking for a solution in decision rules (or closed-loop strategies).

I assume that the level of player i's accumulated lobbying effort required to elicit commitment to player i's cause is stochastic with distribution $F_i(z)$. (Both players have the same subjective probability distribution functions F_1, F_2). Clearly the target can commit its vote to only one side; once committed, the game is over as far as that particular target is concerned.

Retain T as the scheduled date of the vote or decision. Again, let $\rho(z_1(T), z_2(T))$ be the probability that the target votes in support of player 1 at T (given that it has remained uncommitted until that time). Let P_1 be the present value of the gain to player 1 (and P_2 the present value of the loss to player 2) if the target commits itself to vote for the measure. If the target commits itself to player 2's side, then both sides receive a payoff of zero. Finally, let r_i denote player i's discount rate.

With these definitions, we can summarize the players' payoffs:

$$J^1(u_1,u_2) = P_1 \int_0^T (1 - F_2(z_2))f_1(z_1)g^1(u_1,z_1)dt \tag{9a}$$
$$+ P_1(1 - F_1(z_1(T)))(1 - F_2(z_2(T)))\rho(z_1(T),z_2(T))$$
$$- \int_0^T e^{-r_1 t}(1 - F_1(z_1))(1 - F_2(z_2))u_1 dt,$$

$$J^2(u_1,u_2) = -P_2 \int_0^T (1 - F_2(z_2))f_1(z_1)g^1(u_1,z_1)dt \tag{9b}$$
$$- P_2(1 - F_1)z_1(T)))(1 - F_2(z_2(T)))\rho(z_1(T),z_2(T))$$
$$- \int_0^T e^{-r_2 t}(1 - F_1(z_1))(1 - F_2(z_2))u_2 dt.$$

That is, player 1 gains P_1 (2 loses P_2) if either:

a) the target makes a prior commitment to 1 -- this occurs with probability $\int_0^T (1 - F_2)f_1 g^1 dt$; or

b) the target makes no prior commitments but votes in favor at T -- this occurs with probability $(1 - F_1(z_1(T)))$ x $(1 - F_2(z_2(T)))\rho(z_1(T),z_2(T))$. Lobbying expenditures continue only so long as the target is uncommitted (with probability $(1 - F_1(z_1(t)))(1 - F_2(z_2(t))))$.

We seek a Nash equilibrium for the dynamic game described above. A pair $(u_1^N(t,z),u_2^N(t,z))$ is a Nash equilibrium for the dynamic game if

a) $u_i^N(t,z) \in U_i$, $i = 1,2$;

b) u_1^N maximizes $J^1(u_1,u_2^N)$ subject to $\dot{z}_1 = g^1(u_1,z_1)$, $\dot{z}_2 = g^2(u_2^N,z_2)$, $z_i(0) = 0$, $i = 1,2$; and

c) u_2^N maximizes $J^2(u_1^N,u_2)$ subject to $\dot{z}_1 = g^1(u_1^N,z_1)$; $\dot{z}_2 = g^2(u_2,z_2)$; $z_i(0) = 0$, $i = 1,2$.

As usual, define $V^i(t,z)$ to be the maximized value of $J^i(u_i,u_j^N)$ subject to the appropriate constraints, beginning from the initial point (t,z_1,z_2). Then at a Nash equilibrium, if one exists, the pair (u_1^N,u_2^N) must solve [Starr and Ho (1969)]:

$$0 = V_t^1 + \max_{u_1(t,z)\geq 0} \left[V_{z_1}^1 g^1 + V_{z_2}^1 g^2 + L^1(t,z_1,z_2,u_1,u_2)\right] \tag{10a}$$

$$0 = V_t^2 + \max_{u_2(t,z)\geq 0} \left[V_{z_1}^2 g^1 + V_{z_2}^2 g^2 + L^2(t,z_1,z_2,u_1,u_2)\right] \tag{10b}$$

where $L^i(t,z,u)$ denotes the integrand from i's payoff function. In addition, the definition of $V^i(t,z)$ implies the terminal conditions

$$V^1(T,z(T)) = P_1(1 - F_1(z_1(T)))(1 - F_2(z_2(T)))\rho(z_1(T),z_2(T)), \tag{11a}$$
$$V^2(T,z(T)) = -P_2(1 - F_1(t_1(T)))(1 - F_2(z_2(T)))\rho(z_1(T),z_2(T)). \tag{11b}$$

V Example for Opposing Lobbyists

It is extremely difficult to determine any properties of the Nash equilibrium without actually solving a differential game. Accordingly, I use the following special case for illustration:

a) $F_i(z) = 1 - e^{-\lambda z}$

b) $\rho(z(T)) = 1/2$

c) $r_1 = r_2 = r$

d) $g^i(u_i,z_i) = u_i^{1/2}$

e) $P_1 = P_2 = P$

These are basically symmetry assumptions: both players share the same beliefs regarding the target's probability of commitment. If the target makes no prior commitments, then each side is equally likely to receive the target's vote. The players discount at the same rate and are equally adept at producing lobbying effort. Finally, we assume that the result of a commitment to player 1 is a direct wealth transfer of \$P from player 2 to player 1.

We will find it convenient to integrate the first term of J^1 by parts:

$$P\int_0^T (1 - F_2)f_1g^1 = PF_1(z_1(T))(1 - F_2(z_2(T)) + P\int_0^T F_1(z_1)f_2(z_2)g^2 dt.$$

For our example, the Bellman equations are:

$$0 = V_t^1 + \max_{u_1 \geq 0}\left[V_{z_1}^1\sqrt{u_1} + V_{z_2}^1\sqrt{u_2^N} + P\left(1 - e^{-\lambda z_1}\right)e^{-\lambda z_2}\lambda\sqrt{u_2^N} - e^{-rt}e^{-\lambda(z_1+z_2)}u_1\right] \quad (12a)$$

$$0 = V_t^2 + \max_{u_2 \geq 0}\left[V_{z_1}^2\sqrt{u_1^N} + V_{z_2}^2\sqrt{u_2} - P\,e^{-\lambda(z_1+z_2)}\lambda\sqrt{u_1^N} - e^{-rt}e^{-\lambda(z_1+z_2)}u_2\right] \quad (12b)$$

and the terminal conditions are

$$V^1(T,z(T)) = P\left(1 - e^{-\lambda z_1(T)}\right)e^{-\lambda z_2(T)} + \frac{P}{2}e^{-\lambda(z_1(T)+z_2(T))} \quad (13a)$$

$$V^2(T,z(T)) = -\frac{P}{2}e^{-\lambda(z_1(T)+z_2(T))} \quad (13b)$$

The candidate strategies in feedback form are defined by

$$\sqrt{u_1} = \left(V_{z_1}^2 e^{rt}e^{\lambda(z_1+z_2)}\right)/2$$

$$\sqrt{u_2} = \left(V_{z_1}^2 e^{rt}e^{\lambda(z_1+z_2)}\right)/2.$$

Substituting these into (12) yields the Hamilton-Jacobi system:

$$0 = V_t^1 + \frac{(V_{z_1}^1)^2 e^{rt}e^{\lambda(z_1+z_2)}}{4} + \frac{V_{z_2}^1 V_{z_2}^2 e^{rt}e^{\lambda(z_1+z_2)}}{2} + \frac{P\lambda V_{z_2}^2 e^{rt}e^{\lambda z_1}}{2} - \frac{P\lambda V_{z_2}^2 e^{rt}}{2} \quad (14a)$$

$$0 = V_t^2 + \frac{(V_{z_2}^2)^2 e^{rt}e^{\lambda(z_1+z_2)}}{4} + \frac{V_{z_1}^2 V_{z_1}^1 e^{rt}e^{\lambda(z_1+z_2)}}{2} - \frac{P\lambda V_{z_1}^1 e^{rt}}{2} \quad (14b)$$

The form of the terminal conditions, along with symmetry considerations, suggests a solution of the form

$$V^1(t,z) = a(t)e^{-\lambda z_2(T)} + b(t)e^{-\lambda(z_1(T)+z_2(T))},$$

$$V^2(t,z) = b(t)e^{-\lambda(z_1(T)+z_2(T))}.$$

The Hamilton-Jacobi system (14) reduces to

$$0 = \left[\dot{a} + \frac{\lambda^2 be^{rt}(a - P)}{2}\right]e^{-\lambda z_2} + \left[\dot{b} + \frac{b^2 3\lambda^2 e^{rt}}{4} + \frac{bP\lambda^2 e^{rt}}{2}\right] e^{-\lambda(z_1+z_2)} \tag{15a}$$

$$0 = \left[\dot{b} + \frac{b^2 3\lambda^2 e^{rt}}{4} + \frac{bP\lambda^2 e^{rt}}{2}\right] e^{-\lambda(z_1+z_2)} \tag{15b}$$

Since neither $e^{-\lambda z_2}$ nor $e^{-\lambda(z_1+z_2)}$ are zero (for finite z_i), it must be that

$$\dot{b} + \frac{b^2 3\lambda^2 d^{rt}}{4} + \frac{bP\lambda^2 e^{rt}}{2} = 0, \; b(T) = -\frac{P}{2}, \tag{16b}$$

$$\dot{a} + \frac{\lambda^2 be^{rt}(a - P)}{2} = 0, \; a\,(T) = P. \tag{16b}$$

System (16) has solution

$$a(t) \equiv P \quad \forall\, t \in [0,T]$$

$$b(t) = \frac{-2P}{3 + \exp\left\{\frac{P\lambda^2(e^{rt} - e^{rT})}{2r}\right\}}.$$

Since the pair of value functions

$$V^1(t,z) = Pe^{-\lambda z_2} - \frac{2Pe^{-\lambda(z_1+z_2)}}{3 + \exp\left\{\frac{P\lambda^2(e^{rt} - e^{rT})}{2r}\right\}}$$

$$V^2(t,z) = \frac{2Pe^{-\lambda(z_1+z_2)}}{3 + \exp\left\{\frac{P\lambda^2(e^{rt} - e^{rT})}{2r}\right\}}$$

are continuously differentiable and since they and the stategies

$$u_i^N = \left(V^i_{z_i} e^{rt} e^{\lambda(z_1+z_2)}\right)^2 \Big/ 4 \qquad i = 1,2 \tag{17}$$

solve the system (13-14), by the verification theorem [Stalford and Leitmann (1973, Theorem 1)], (u_1^N, u_2^N) as defined in (17) constitutes a Nash equilibrium in pure strategies.

Thus we have proven:

Proposition 3: There exists a symmetric Nash equilibrium in pure (closed-loop) strategies. It depends only on t and

$$u_i^N(t,z) = \left[\frac{2P\lambda e^{rt}}{3 + \exp\left\{\frac{P\lambda^2(e^{rt} - e^{rT})}{2r}\right\}}\right]^2, \qquad i = 1,2$$

The Nash equilibrium payoffs are

$$V^1(0,0) = P - \frac{2P}{3 + \exp\left\{\frac{P\lambda^2(1 - e^{rT})}{2r}\right\}},$$

$$V^2(0,0) = \frac{-2P}{3 + \exp\left\{\frac{P\lambda^2(1 - e^{rT})}{2r}\right\}} .$$

We find that the results of Proposition 1 for the unopposed lobbyist are immediately invalidated for the case of strategic lobbyists.

Proposition 4: The Nash equilibrium strategies may be

a) everywhere increasing in t;

b) everywhere decreasing in t;

c) first increasing, reaching a single peak, then decreasing in t.

Proof: $\dot{u}^N = \frac{2rM^2}{D^3}\left[3 + \frac{e^m(1 - P\lambda^2e^{rt})}{2r}\right]$

where M and D are the numerator and denominator, respectively, in the expression for u_1^N; and $m(t) = P\lambda^2(e^{rt} - e^{rT})/2r$. Thus, sgn $\dot{u}^N$ = sgn$\left[3 + e^m(1 - P\lambda^2e^{rt})/2r\right]$.

Let us examine the function $h(t) = 3 + e^m(1 - P\lambda^2e^{rt}/2r)$.

Since $h'(t) = -\frac{e^m}{r}\left(\frac{P\lambda^2e^{rt}}{2}\right)^2 < 0$, h(t) is greatest at t = 0 and smallest at t = T. Thus:

a) if $h(T) = 4 - \frac{P\lambda^2e^{rT}}{2r} \geq 0$, then $\dot{u}^N > 0 \quad \forall\, t \in [0,T)$ (with $\dot{u}^N(T) \geq 0$).

b) if $h(0) = 3 + \exp\left\{P\lambda^2 \frac{(1 - e^{rT})}{2r}\right\}\left(1 - \frac{P\lambda^2}{2r}\right) \leq 0$,

then $\dot{u}^N < 0 \quad \forall\, t \in (0,T]$ (with $\dot{u}^N(0) \leq 0$).

c) if h(T) < 0, and h(0) > 0 (this covers all remaining cases), then u^N is first increasing, attains a single peak where h(t) = 0, and thereafter declines.

Thus we could observe a decreasing rate of expenditure on lobbying for the case of strategic lobbyists, while an unopposed lobbyist always spends at an increasing rate.

Comparative statics results are summarized in Proposition 5 and proved in the Appendix.

Proposition 5:

$$\frac{\partial u^N}{\partial P} > 0 \quad \forall\, t \in [0,T].$$

$$\frac{\partial u^N}{\partial \lambda} > 0 \quad \forall\, t \in [0,T]$$

$$\frac{\partial u^N}{\partial T} > 0 \quad \forall\, t \in [0,T]$$

$$\frac{\partial u^N}{\partial r} > 0 \quad \forall\, t \in [0,T] \text{ (so long as } T > 1/r).$$

It appears that an increase in any parameter is sufficient to generate an increase in the Nash equilibrium rate of lobbying expenditures. P represents the amount of the wealth transfer from 1 to 2 in the event of the target's commitment to side 1. Thus the lobbyist increases its expenditures in order to obtain the larger wealth transfer while the counterlobby increases its expenditures in order to prevent the larger loss. Recall that $1/\lambda$ is the expected level of accumulated lobbying effort required to obtain commitment to each one of the sides. As λ increases, this mean required effort decreases, so we have the result that targets with lower mean required effort will be more strenuously lobbied. This contrasts with the unopposed lobbyist case wherein an increase in λ results in a redistribution of expenditure from the near to the distant future. An increase in the scheduled vote date T also results in increased lobbying expenditure. This is in direct conflict with the result of Proposition 2-(iii) for the unopposed lobbyist. Finally, an increase in the discount rate also results in a uniform increase in the rate of lobbying expenditure, again in contrast to Proposition 2-(v), which involves redistribution from the near to the distant future.

Since the game discussed in this section would appear to be the natural extension of the model of section III, we can only attribute these differences to the existence of a strategic counterlobby.

VI Optimality

This game has the well-known prisoner's dilemma structure. For example, at the Nash equilibrium, both players are worse off than if they had simply saved their money and taken their chances at T. Unless we assume that the members of the lobbying organizations derive utility from spending money on a cause they believe in irrespective of the dollar payoffs (which may in part

be true), we are faced with the task of explaining why noncooperative lobbying occurs. One reason could be as follows: even if the two opponents agree on a compromise position, they have no power to enact laws or issue directives to enforce their decision. They must still go through the decision-makers who are empowered to enforce their solutions. These decision-makers appear to be the only agents who benefit from excessive lobbying behavior. Thus they have a vested interest in keeping the lobbies at each other's throats. This suggests that further modeling of the target's behavior is required. In particular, we would be interested in the extent to which the targets themselves possess the power and incentive to create and maintain such prisoner's dilemma games.

VII Discussion

This model is obviously an extreme oversimplification of what occurs in practice. In particular, it models only a single target. In reality, the commitment of a specific target may precipitate the commitment of many others. Thus the value of a target's commitment depends in part on how many others are likely to follow suit. Furthermore, the value of a target's commitment depends upon how many commitments have already been collected. For example, any votes over a majority may be worthless. Thus there is really a sequential aspect to the problem as well. The irrevocability assumption is contrived; the decision to renege should be part of the modeling of the target's equilibrium behavior. This equilibrium target behavior could be modeled in a Stackelberg framework, with the target as leader, inasmuch as the target is empowered to alter institutions which the populace in general must take as given, at least until it ousts the target.

Since I have not explicitly modeled the target's behavior, it is not clear why the target could ever be expected to commit itself. That is, why is it not optimal for the target to extract maximum lobbying benefits and then simply cast its vote with the side which minimizes its political costs? I suggest that there will exist an optimal level of lobbying effort (after which the target ought to commit itself) because the game is repeated on subsequent issues. If the target is observed never to commit before T, then future lobbying expenditures will be reduced (as $E[z_i]$ is revised upward). Thus there is some tradeoff between current and long-term gains, suggesting that there should exist some reservation level of lobbying effort which triggers commitment. This commitment then acts as a signal to future lobbyists that the target is willing to be persuaded.

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APPENDIX

Proof of Proposition 2:

Let $D = 4 - P(1 - \rho)\lambda^2 \frac{(e^{rt} - e^{rT})}{r}$ and $N = 2P(1 - \rho)\lambda e^{rt}$.

i) $\frac{\partial n^*}{\partial P} = \frac{8N^2}{PD^3} > 0 \quad \forall\ t \in [0,T].$

ii) $\frac{\partial u^*}{\partial \rho} = \frac{-8N^2}{(1 - \rho)D^3} < 0 \quad \forall\ t \in [0,T].$

iii) $\frac{\partial u^*}{\partial T} = \frac{-2N^2}{D^3} [P(1 - \rho)\lambda^2 d^{rT}] < 0 \quad \forall\ t \in [0,T].$

iv) $\frac{\partial u^*}{\partial \lambda} = \frac{2N^2}{\lambda D^3} \left[4 + P(1 - \rho)\lambda^2 \frac{(e^{rt} - e^{rT})}{r}\right]$

Thus sgn $\frac{\partial u^*}{\partial \lambda}$ = sgn $4 + P(1 - \rho)\lambda^2 \frac{(e^{rt} - e^{rT})}{r}$. So $\frac{\partial u^*}{\partial \lambda} \gtrless 0$ as

$t \gtrless \hat{t} = \frac{1}{r} \ln\left\{e^{rT} - \frac{4r}{P(1 - \rho)\lambda^2}\right\}.$

v) $\frac{\partial u^*}{\partial r} = \frac{2N^2}{D^3} \left[4t + P(1 - \rho)\lambda^2 e^{rT} \frac{(t - T)}{r} - P(1 - \rho)\lambda^2 \frac{(e^{rt} - e^{rT})}{r^2}\right]$

Thus sgn $\frac{\partial u^*}{\partial r}$ = sgn $g(t)$ where

$$g(t) = 4t + P(1 - \rho)\lambda^2 \left[e^{rT} \frac{(t - T)}{r} - \frac{(e^{rt} - e^{rT})}{r^2}\right].$$

Since $g(0) < 0$, $g(T) > 0$, and $g^1(t) > 0$, $\frac{\partial u^*}{\partial r}$ is first negative, becomes zero for some $\tilde{t} \in (0,T)$, and then is positive until T.

Proof of Proposition 5:

Let $M = 2P\lambda e^{rt}$ and $D = 3 + e^m$, where $m(t) = P\lambda^2 \frac{(e^{rt} - e^{rT})}{2r}$

i) $\frac{\partial u^N}{\partial P} = \frac{2M^2}{PD^3} [3 + e^m(1 - m)] > 0 \quad \forall\ t \in [0,T].$

ii) $\frac{\partial u^N}{\partial \lambda} = \frac{2M^2}{\lambda D^3} [3 + e^m(1 - 2m)] > 0 \quad \forall\ t \in [0,T].$

iii) $\frac{\partial u^N}{\partial T} = \frac{M^2}{D^3} [e^m P\lambda^2 e^{rT}] > 0 \quad \forall\ t \in [0,T].$

iv) $\frac{\partial u^N}{\partial r} = \frac{2M^2}{D^3} \left[3t + te^m + \frac{e^m P\lambda^2(e^{rt}(1/r - t) + e^{rT}(T - 1/r))}{2r}\right]$

Let $g(t) = e^{rt}(1/r - t) + e^{rT}(T - 1/r)$. Now $g(0) = 1/r + e^{rT}(T - 1/r) > 0$ (for $T \geq 1/r$); $g(T) = 0$ and $g^1(t) = -tre^{rt} < 0$ for $t > 0$.

Thus $g(t) \geq 0 \quad \forall\ t \in [0,T]$ and $3t + te^m + \frac{e^m P\lambda^2 g(t)}{2} > 0 \quad \forall\ t \in [0,T].$